

Power and Exponential Calibrations



Definition - Power

Power is the number of times, as indicated by an exponent, that a number occurs as a factor in a product.

*Ref: Merriam-Webster Online Dictionary. Power.
Retrieved on October 5, 2006 from <http://www.m-w.com/dictionary/power>*



Power Equation

$$y = ax^b$$

Where: y = response
 x = concentration
 a = intercept
 b = slope

Ref: NIST/SEMATECH e-Handbook of Statistical Methods, <http://www.itl.nist.gov/div898/handbook/>, Section 2.3.6.1, Retrieved September 8, 2006.



Power Equation Transformed Into Linear Equation

$$y = ax^b$$

$$\log y = b \log x + \log a$$

Where:

y = response

x = concentration

a = intercept

b = slope

*Ref: Zar, Jerrold H. 1996. Biostatistical
Analysis: Third Edition. Chapter 19: Multiple
Regression and Correlation, page 434.
Published by Prentice Hall*



Definition - Exponential

Exponential is where a variable relates to an exponent (example 10^x) and is characterized by an extremely rapid increase.

*Ref: Merriam-Webster Online Dictionary.
Exponential. Retrieved on October 5, 2006 from
<http://www.m-w.com/dictionary/exponential>*



Exponential Equation

$$y = ae^{bx}$$

Where: y = response
 x = concentration
 a = intercept
 b = slope
 e = exponential

*Refs: The KaleidaGraph Guide to Curve Fitting,
Retrieved on October 2, 2006 from*

www.synergy.com/Tools/curvefitting.pdf

*The Industrial Physicist: Primer. Retrieved on
October 2, 2006 from*

<http://www.tipmagazine.com/tip/INPHFA/vol-9/iss-2/p24.html>



Exponential Equation Transformed Into Linear Equation

$$y = ae^{bx}$$

$$\ln y = \ln a + bx$$

or

$$\log y = (bx/2.303) + \log a$$

Where: y = response
 x = concentration
 a = intercept
 b = slope

*Ref: Zar, Jerrold H. 1996. Biostatistical
Analysis: Third Edition. Chapter 19: Multiple
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Regression Statistical Equations

When solving the slope, intercept, and correlation for a power or exponential calibration that has been transformed into a linear least square regression, the analyst can follow the equations as described for a linear least square regression. Be aware that the natural logarithm and the logarithm components need to be carried through the equations.



Transformations

Data is typically transformed when heteroscedasticity (nonequal variances in the data) occurs.

- An instance when the logarithmic transformation is applicable is when there is heteroscedasticity and the standard deviations are proportional to the means (i.e. there is a constant coefficient of variation)



Transformations-Cont.

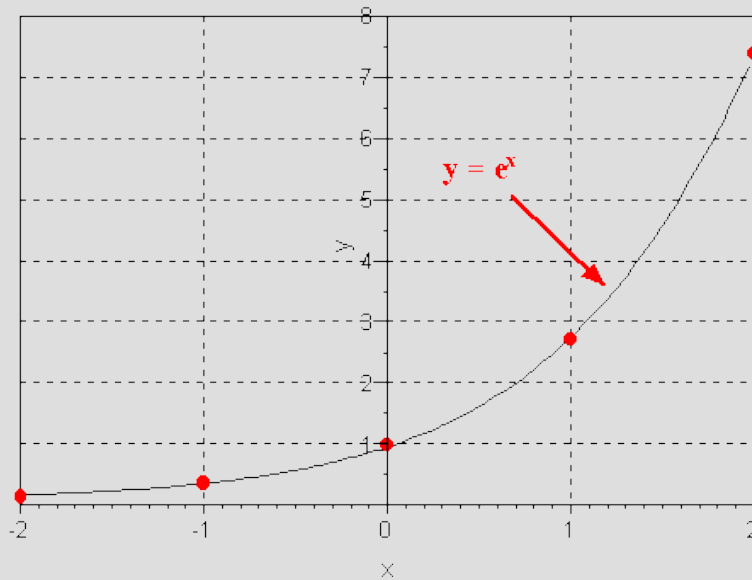
- Another instance to use logarithmic transformation is when the effects in an analysis of variance are multiplicative rather than additive, then the logarithms of the data will exhibit additivity

Refs: Kelley, William D.; Ratliff, Jr., Thomas, A.; and Nenadic, Charles. (1992) Basic Statistics for Laboratorians: A Primer for Laboratory Workers, page 86. Published by John Wiley & Sons, Inc

Zar, Jerrold H. 1996. Biostatistical Analysis: Third Edition. Chapter 13: Data Transformations, page 279. Published by Prentice Hall

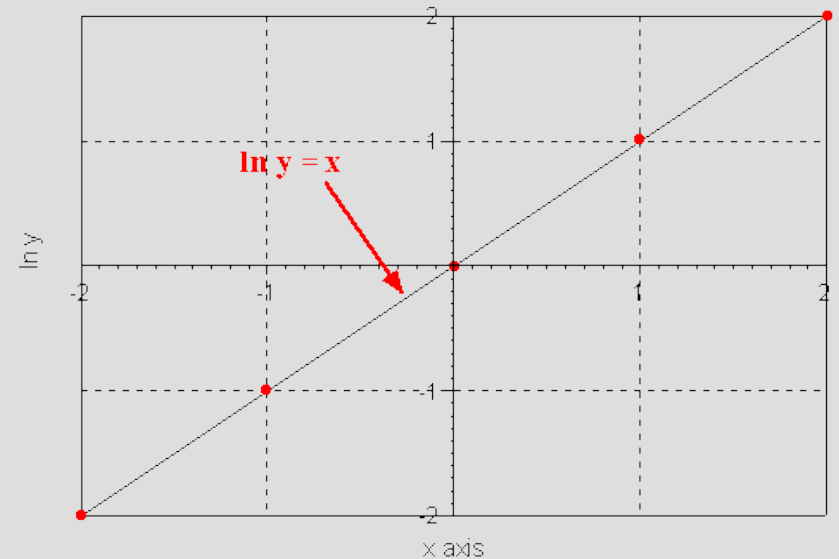


Figure 3: Exponential Equation (non-linear form)



← Curve Before Transformation

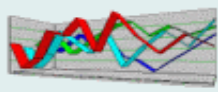
Figure 4: Transformed Equation (linear form)



Curve After Transformation →

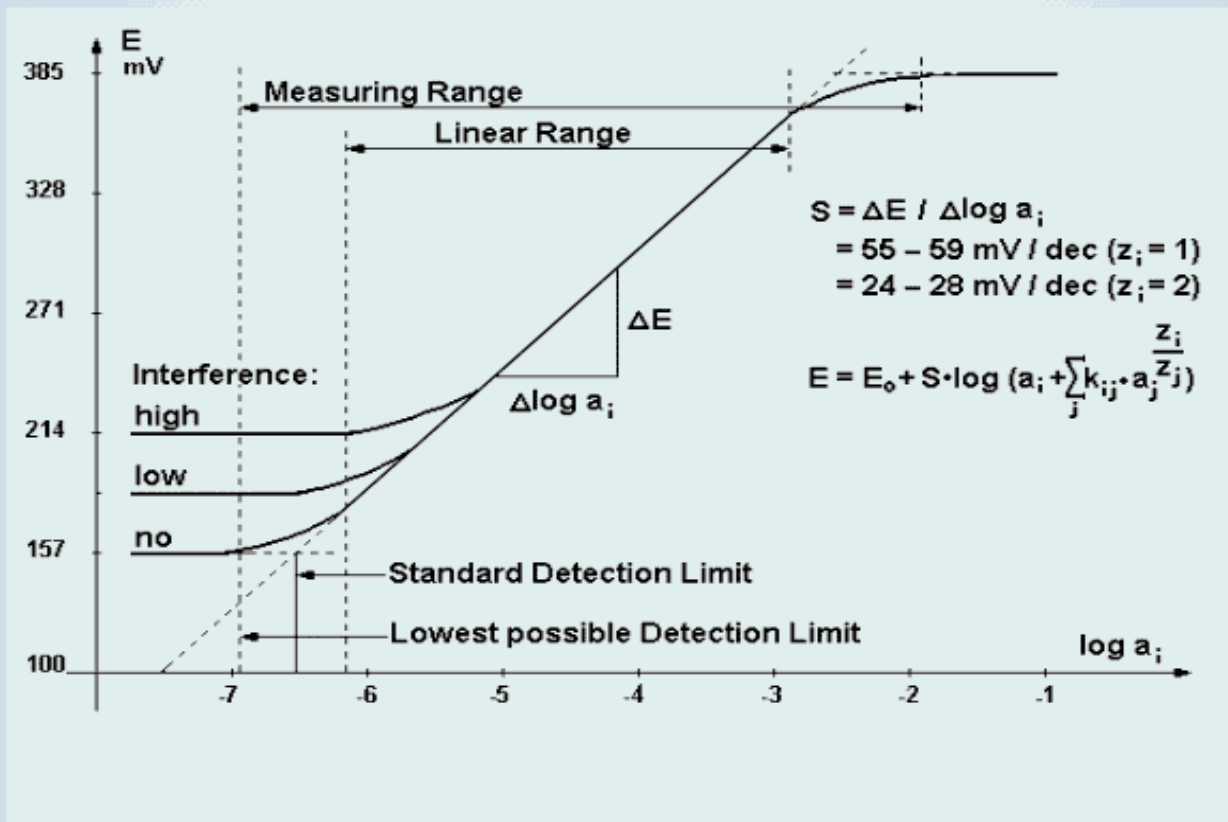
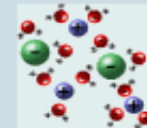
Ref: Donahue, Craig. Edited By: The Science Learning Center Staff. Transformations of Exponential Equations. Retrieved on October 2, 2006 from

<http://www.umd.umich.edu/casl/natsci/slc/slconline/TRANSF/index.html>



Typical ISE Calibration Graph

NB: X-axis units are the logarithm of the Molar Activity of the Ion



NICO 2000 LTD.

Example of a Curve Using Logarithmic Concentration

Ref: Beginners Guide to ISE Measurement. Chapter 7: Calibration Theory. Retrieved September 12, 2006 from <http://www.nico2000.net/Book/Guide8.html>



Limitations

Exponential calibration curves require a minimum of two calibration standards.

Ref: DIONEX Chromeleon Software. Help: Calibration Type. Software version 6.60

Power or exponent calibration curves require a minimum of two calibration standards when the origin is included and three calibration standards when the origin is ignored.

Ref: HP 3365 Series II Chemstation: Reference Manual Volume II, Chapter 16: Report Calculations, page 16-13



Limitations

Power and exponential curves are used to fit data that increases or decreases at a high rate. Neither curve can fit negative data or data equal to zero.

*Ref: The KaleidaGraph Guide to Curve Fitting,
Retrieved on October 2, 2006 from
www.synergy.com/Tools/curvefitting.pdf*



Limitations

Use of the Power calibration requires an understanding of detector behavior and a data transformation that will allow a simple calibration model.

*Ref: SW-846 Method 8000C: Determinative
Chromatographic Separations: Section 11.5.4
Retrieved on September 25, 2006 from
<http://www.epa.gov/epaoswer/hazwaste/test/new-meth.htm#8000C>*



Benefits of Power/Exponential

- Transformation of data into simpler calibration model
- Reflects behavior of detector/electrode better
- A useful approximation of the average of the actual changing rate during respective time durations

*Ref: SW-846 Method 8000C: Determinative
Chromatographic Separations: Section 11.5.4
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Disadvantages of Power/Exponential

Transformation of data may result in data not meeting assumptions of normality and homoscedasticity (equal variances)

Refs: Kelley, William D.; Ratliff, Jr., Thomas, A.; and Nenadic, Charles. (1992) Basic Statistics for Laboratorians: A Primer for Laboratory Workers, page 86. Published by John Wiley & Sons, Inc.

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